Mark scheme for Extension Worksheet – Topic 2, Worksheet 7

1 **a** $\frac{GM^2}{(2a)^2} = \frac{Mv^2}{a}$

for each correct side of the equation. The result follows after cancellations. [2]

b Use of $v = \frac{2\pi a}{T}$; combine with $v^2 = \frac{GM}{4a}$; to get answer. [2]

c $E_T = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 - \frac{GM^2}{2a}$; substitute the expression for speed to get $E_T = \frac{1}{2}M\frac{GM}{4a} + \frac{1}{2}M\frac{GM}{4a} - \frac{GM^2}{2a}$; simplify to get answer, $E_T = -\frac{GM^2}{4a}$ [2]

d The total energy of the stars will be reduced and from $E_T = -\frac{GM^2}{4a}$ it follows that (i) orbit radius must become smaller; from $v^2 = \frac{GM}{4a}$ it follows that (ii) the speed will increase; from $T = 4\pi\sqrt{\frac{a^3}{GM}}$ (iii) period will decrease. [3]

2 **a** Application of energy conservation: $\frac{1}{2}mv^2 + \frac{kQq}{d} = \frac{kQq}{R}$; manipulation to get charge: $\frac{1}{2}mv^2 = Qkq(\frac{1}{R} - \frac{1}{d}) \Rightarrow Q = \frac{mv^2}{2kq(\frac{1}{R} - \frac{1}{d})}$; substitution to get

$Q = \frac{0.020 \times (2.5 \times 10^2)^2}{2 \times 8.99 \times 10^9 \times 45 \times 10^{-6}(\frac{1}{0.25} - \frac{1}{0.85})}$; to give $Q = 547 \approx 550\ \mu\text{C}$ [3]

b $E = \frac{kQ}{R^2} = \frac{8.99 \times 10^9 \times 547 \times 10^{-6}}{0.25^2}$; $E = 7.8 \times 10^7\ \text{N C}^{-1}$ [2]

c The charge will move radially away from the sphere; with decreasing acceleration. [2]

d Application of energy conservation: $\frac{1}{2}mv^2 = \frac{kQq}{R}$; solving for the speed:

$v = \sqrt{\frac{2kQq}{mR}} = \sqrt{\frac{2 \times 8.99 \times 10^9 \times 547 \times 10^{-6} \times 45 \times 10^{-6}}{0.020 \times 0.25}} = 8.9 \times 10^4\ \text{m s}^{-1}$ [2]